Graph Theory

Practice exam 2

Problem 1. Let G be a bipartite graph with parts $A = \{a_1, \ldots, a_n\}$ and B. Suppose that $d(a_i) = i$ for every $1 \le i \le n$. Show that G has a matching covering A.

Problem 2. Let $x_1, \ldots, x_n$ be irrational numbers. Show that there are at most $\lfloor \frac{n^2}{4} \rfloor$ pairs $1 \le i < j \le n$ such that $x_i + x_j \in \mathbb{Q}$.

Problem 3. Let G be a graph on n vertices with at least $2n-2$ edges. Show that G contains two cycles of the same length.

Problem 4. Let G be a graph with $\chi(G) = k$.

(a) Show that $e(G) \ge \binom{k}{2}$.

(b) Suppose further that $e(G) = \binom{k}{2}$ and G has no isolated vertices. Show that $G = K_k$.

Problem 5. Let G be a $(k+1)$-connected graph, and let $a, b, x_1, \ldots, x_k$ be distinct vertices in G. Show that there is a path from a to b containing all vertices $x_1, \ldots, x_k$.

Problem 6. Let G be a graph on $n \ge 6$ vertices with minimum degree at least $n/2$. Prove that there exist two vertex-disjoint cycles in G which together cover the vertex set of G. For the purpose of this problem, we consider a single vertex and a single edge to be cycles.