Graph Theory

Instructor: Oliver Janzer

Assignment 2

This exercise sheet contains problems relating to Section 2 of the lecture notes

Please submit your solution to Problem 2 by the end of September 23rd for feedback.

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Show that in a tree containing an even number of edges, there is at least one vertex with even degree.

Problem 2: Given a graph G and a vertex $v \in V(G)$, $G - v$ denotes the subgraph of G induced by the vertex set $V(G) \setminus \{v\}$. Show that every connected graph G of order at least two contains vertices x and y such that both $G - x$ and $G - y$ are connected.

Problem 3: Let T be a tree with exactly $2k$ odd-degree vertices. Prove that T decomposes into k paths (i.e. its edge-set is the disjoint union of k paths).

Problem 4: Prove that a connected graph G is a tree if and only if any family of pairwise (vertex-)intersecting paths $P_1, \dots, P_k$ in G have a common vertex.