Graph Theory

Instructor: Oliver Janzer

Assignment 3

Please submit your solution to Problem 3 by the end of September 30th for feedback.

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1:

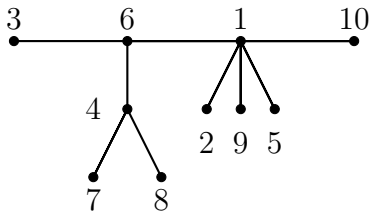
(a) Describe which Prüfer codes correspond to stars (i.e. to trees isomorphic to $K_{1,n-1}$).

(b) Describe what trees correspond to Prüfer codes containing exactly 2 different values.

Problem 2: Let T be a forest on vertex set $[n]$ with components $T_1, \dots, T_r$. Prove, by induction on r, that the number of spanning trees on $[n]$ containing T is $n^{r-2} \prod_{i=1}^{r} |T_i|$. Deduce Cayley's formula.

Problem 3:

(a) What is the Prüfer code of the following tree? What is the map associated with it in Joyal's proof with left end 4 and right end 5?



(b) Which labeled tree has Prüfer code (5,1,1,7,7,5)?