Graph Theory

Instructor: Oliver Janzer

Assignment 5

Please submit your solution to Problem 1 by the end of October 14th for feedback

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Prove that a graph G with at least 3 vertices is 2-connected if and only if for any three vertices x, y, z there is a path from x to z containing y.

Problem 2: Let G be a k-connected graph, where $k \geq 2$. Show that if $|V(G)| \geq 2k$ then G contains a cycle of length at least $2k$.

[Hint: .ti dnetxe ot yrt dna htgnel mumixam fo elcyc a ekaT]

Problem 3: A matching is a set of pairwise-disjoint edges. Let G be a bipartite graph, and suppose that G has no matching of size k. Prove that there is a set $X \subseteq V(G)$, $|X| \leq k-1$, such that X intersects every edge of G. This statement is called König's theorem.

Problem 4: Give a complete proof of Corollary 3.18 (ii) from the notes. That is, show that for every graph G and distinct vertices u, v, the minimum number of edges separating u from v in G is equal to the maximum number of edge-disjoint u-v-paths in G.