Graph Theory

Instructor: Oliver Janzer

Assignment 9

Please submit your solution to Problem 1 by the end of November 18th for feedback.

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Determine all positive integers r and s, with $r \leq s$, for which $K_{r,s}$ is planar.

Problem 2:

(a) Show that every planar graph has a vertex of degree at most 5. Is there a planar graph with minimum degree 5?

(b) Show that any planar *bipartite* graph has a vertex of degree at most 3. Is there a planar bipartite graph with minimum degree 3?

Problem 3: Show that a connected plane graph G is bipartite iff all its faces have even length.

Problem 4: Let G be a graph on $n \geq 3$ vertices and $3n - 6 + k$ edges for some $k > 0$. Show that any drawing of G in the plane contains at least k crossing pairs of edges.

Problem 5: Let G be a plane graph with triangular faces and suppose the vertices are colored arbitrarily with three colors. Prove that there is an even number of faces that get all three colors.

[Hint: .syaw tnereffid owt ni segde deroloc-2 fo rebmun eht tnuoC]

Note: We remark that the outer face of G should also be a triangle.

Problem 6: Let S be a set of $n \geq 3$ points in the plane such that any two of them have distance at least 1. Show that there are at most $3n - 6$ pairs of distance *exactly* 1.

[Hint: .ytilauqeni elgnairt eht gnisu gnissorc on sah hparg eht evorP]