Graph Theory

Instructor: Oliver Janzer

Assignment 10

Please submit your solution to Problem 4 by the end of November 25th for feedback.

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Are the following statements true?

(a) If G and H are graphs on the same vertex set, then $\mathrm{dg}(G \cup H) \leq \mathrm{dg}(G) + \mathrm{dg}(H)$.

(b) If G and H are graphs on the same vertex set, then $\chi(G \cup H) \leq \chi(G) + \chi(H)$.

(c) Every graph G has a $\chi(G)$-coloring where $\alpha(G)$ vertices get the same color.

Problem 2: G has the property that any two odd cycles in it intersect (they have at least one vertex in common). Prove that $\chi(G) \leq 5$.

[Hint: .hparg etitrapib a teg ot secitrev emos evomeR]

Problem 3: For a vertex v in a connected graph G, let G_r be the subgraph of G induced by the vertices at distance r from v. Show that $\chi(G) \leq \max_{0 \leq r \leq n} \chi(G_r) + \chi(G_{r+1})$.

Problem 4: Let l be the length of the longest path in a graph G. Prove $\chi(G) \leq l + 1$ using the fact that if a graph is not d-degenerate then it contains a subgraph of minimum degree at least $d + 1$.

Problem 5: Suppose the complement of G is bipartite. Show that $\chi(G) = \omega(G)$.

[Hint: tnemelpmoc etitrapib sti fo ytreporp a ot G fo ytreporp siht etalsnarT .esruoc eht morf tluser reilrae na esu dna]