Graph Theory

Instructor: Oliver Janzer

Assignment 11

Please submit your solution to Problem 2 by the end of December 2nd for feedback.

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: For a given natural number n, let G_n be the following graph with $\binom{n}{2}$ vertices and $\binom{n}{3}$ edges: the vertices are the pairs (x, y) of integers with $1 \leq x < y \leq n$, and for each triple (x, y, z) with $1 \leq x < y < z \leq n$, there is an edge joining vertex (x, y) to vertex (y, z). Show that for any natural number k, the graph G_n is triangle-free and has chromatic number $\chi(G_n) > k$ provided $n > 2^k$.

[Hint: .tcudni dna $\{k$ roloc sah (y, x) on $: x\}$ dna $\{k$ roloc sah (x, y) on $: x\}$ stes eht enifeD]

Problem 2: A proper edge-coloring of a graph G is an assignment of colors to the edges of G such that no two edges with a common endpoint have the same color. The edge-chromatic number of G is the minimum number of colors in a proper edge-coloring of G. Find the edge-chromatic number of K_n.

[Hint: tsrif n ddo oD]