Graph Theory

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Assignment 12

Please submit your solution to Problem 1 by the end of December 9th for feedback.

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: The lower bound for $R(p,p)$ that you learn in the lectures is not a constructive proof: it merely shows the *existence* of a red-blue coloring not containing any monochromatic copy of K_p.

Give an explicit coloring on $K_{(p-1)^2}$ that proves $R(p,p) > (p-1)^2$.

Problem 2: Show that every red/blue-colouring of the edges of K_{6n} contains n vertex-disjoint triangles with all $3n$ edges of the same colour.

Problem 3:

(a) Let $n \geq 1$ be an integer. Show that any sequence of $N \geq R(n,n)$ distinct numbers, $a_1, \ldots, a_N$ contains a monotone (increasing or decreasing) subsequence of length n.

(b) Let $k, l \geq 1$ be integers. Show that any sequence of $kl+1$ distinct numbers $a_1, \ldots, a_{kl+1}$ contains a monotone increasing subsequence of length $k+1$ or a monotone decreasing subsequence of length $l+1$.

[Hint: .rebmun hcae ta gnitrats ecneuqes gnisaerced tsegnol eht ta kooL
.$a_i < a_j$ neht $i < j$ secidni rof emas eht si htgnel siht fI]