Graph Theory

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Assignment 13

Please submit your solution to Problem 1 by the end of December 16th for feedback.

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Prove that if there is a real number p, $0 \leq p \leq 1$, such that

$$\binom{n}{k} p^{\binom{k}{2}} + \binom{n}{t} (1-p)^{\binom{t}{2}} < 1,$$

then the Ramsey number $R(k,t)$ satisfies $R(k,t) > n$. Using this, show that the following holds, for some constant c.

$$R(4,t) \geq c \cdot \frac{t^{3/2}}{(\log t)^{3/2}}.$$

Problem 2: Prove that for every fixed positive integer r, there is an n such that any coloring of all the subsets of $[n]$ using r colors contains two non-empty disjoint sets X and Y such that X, Y and $X \cup Y$ have the same color.

Problem 3: Prove the following strengthening of Schur's theorem: for every $k \geq 2$ there is an N such that any k-coloring of $[N]$ contains three *distinct* integers a, b, c of the same color satisfying $a + b = c$.

Problem 4: Prove that for every $k \geq 2$ there exists an integer N such that every coloring of $[N]$ with k colors contains three distinct numbers a, b, c satisfying $ab = c$ that have the same color.